Homework Set 13: Driven Harmonic Motion Solution

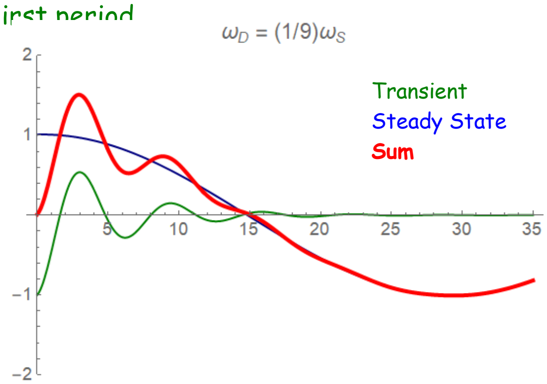
Problem From TM5.

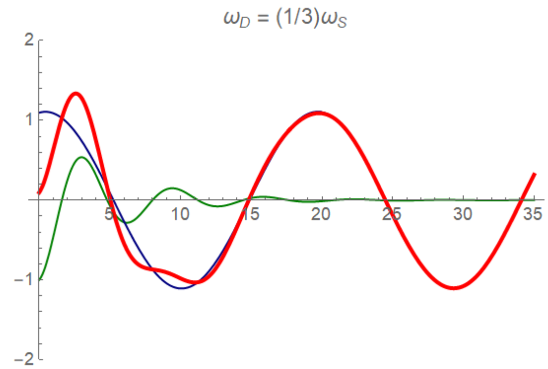
1) 3-24 *Altered* For β = 0.2 s^{-1}, Mathematic plots like those shown in Figure 3-15 for a sinusoidal driven, damped oscillator where $x_p(t)$, $x_c(t)$, and the sum x (t) are displayed on the back of this sheet. To produce them, I let k = 1 kg/s^2, m = 1 kg, A = -1 m, the phase angle δ = 0, and plotted values of ω_D/ω_S of 1/9, 1/3, 1.1, 3 and 6. For the $x_p(t)$ solution (Eqn. 3.60), I let F_0/m = 1 m/s^2, but calculatde δ. For the last plot, in the $x_p(t)$ solution (Eqn. 3.60), I let F_0/m = 20 m/s^2. For ω_D/ω_S = 6, let F_0 = 20 m/s^2 for $x_p(t)$ and produce the plot again.

What do you observe about the relative amplitudes of the two solutions as ω_D increases? Why does this occur?

The amplitude of the steady-state term can be seen to increase as ω_D approaches the value of ω_S. then decrease as ω_D gets much larger than the value of ω_S.

As seen in the ω_D/ω_S = 1/9 and 1/3, the sum is dominated by the transient with a large amplitude, but only in the first period



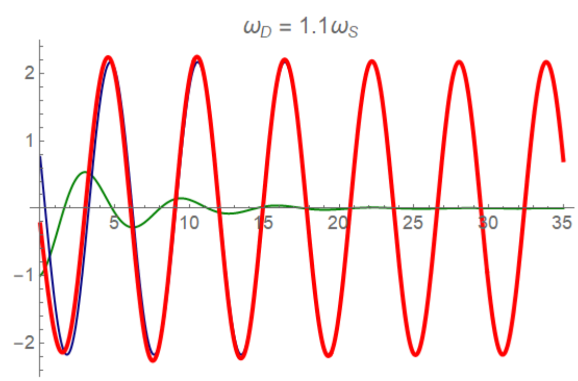


The dependence of the steady-state amplitude on the frequencies, in particular, the difference in the frequencies squared in the denominator, means its amplitude is maximized when the difference is the smallest. This is shown by the ω_D/ω_S = 1.1 plot:

$$H = \frac{F_0/m}{\sqrt{\left(\omega_N^2 - \omega_D^2\right)^2 + 4\beta^2\omega_D^2}}$$

Frequency difference.

$4\beta^2$ makes this term constant and small.



Once ω_D is larger than ω_S, the steady-state amplitude becomes increasingly small due to both terms in the denominator becoming large. In terms of warping, when ω_D is smaller than ω_S, the transient warps the steady state, but only for the first period. When they're nearly equal, the transient has the least effect. When $\omega_S > \omega_D$, the steady state warps the transient, but damps out after 3 periods as shown in the last two plots.

The plots show driven, under dampled harmonic oscillations for

$$x(t) = Ae^{-\beta t}\cos(\omega_S t) + \frac{F_0/m}{\sqrt{\left(\omega_N^2 - \omega_D^2\right)^2 + 4\beta^2\omega_D^2}}\cos(\omega_D t - \delta), \text{ where } \delta = \tan^{-1}\left(\frac{2\beta\omega_D}{\omega_N^2 - \omega_D^2}\right)$$

with $F_0/m = k = 1$, $\delta_{transient} = 0$, $A = -1$, and $\beta = 0.2\ s^{-1}$ (giving $\omega_N = 1\ s^{-1}$ and $\omega_S = 0.9798\ s^{-1}$) and ω_D as multiples of ω_N as given on each plot.